Decision Analysis and Real Options: A Discrete Time Approach to Real Option Valuation

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Abstract

In this paper we seek to enhance the real options methodology originally developed by Copeland and Antikarov (2001) with traditional decision analysis tools to propose a discrete time method that allows the problem to be specified and solved with off the shelf decision analysis software. This method uses dynamic programming with an innovative algorithm to model the project's stochastic process and real options with decision trees. The method is computationally intense, but simpler and more intuitive than traditional methods, thus allowing for greater flexibility in the modeling of the problem.

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1. Introduction

Due to its importance for the creation of value for the shareholder, the investment decision in the firm has always been a focus of academic and managerial interest. The use of the discounted cash flow method (DCF), introduced in firms in the 1950's, was initially considered a sophisticated approach for the valuation of projects due to the need to use present value tables. In spite of its obvious advantages over the obsolete payback method in use until then, its widespread use occurred only after the development of portable calculators and computers that automated the necessary financial calculations, and most practitioners currently consider it the model of choice.

More recently, the pioneering work of Black and Scholes (1973) and Merton (1973) for the evaluation of financial options provided the groundwork for the idea of incorporating option pricing methods into the problem of valuing real investments under uncertainty. These methods add the value of managerial flexibility to the traditional DCF approach, and have been called real options theory, to indicate the focus on options associated with real assets rather than with financial assets. However, despite its theoretical appeal, the mathematical complexity of real options valuation methods has limited the use of this approach by a broader audience in the industry.

This additional complexity is due to several factors. The underlying assets for financial options are usually market securities, commodities, or financial assets that possess characteristics facilitating the valuation of the option. These characteristics include market price, historical data, divisibility and a reasonable knowledge of their probabilistic distributions, which allow one to model their future distributions with some degree of confidence. Real options, on the other hand, are more complex because the real assets that are their underlying assets usually do not have most of these traits. Another source of complexity is the high degree of mathematical sophistication necessary for modeling in continuous time, generally beyond the skills of most practitioners. But, as occurred with the DCF method, the continuing evolution of computational tools to automate the more difficult parts of the process and some progress regarding the underlying theory have tended to make the use of real options techniques more and more widespread.

In this paper we seek to enhance the real options methodology originally developed by Copeland and Antikarov (2001) with traditional decision analysis tools to propose a method that addresses these issues while allowing the problem to be specified and solved with off-the-shelf decision analysis software. We do this by first determining a set of virtual cash flows (pseudo cash flows) and risk neutral probabilities that will give the correct project values when discounted to each period and state. Project flexibilities, or real options, can then be modeled easily as decisions that affect their pseudo cash flows. The specification of the project values in time as a function of future cash flows also allows the problem to be modeled as a decision tree, and allows the use of commercially available decision tree software.

Previous work on the decision analysis perspective on real options has been limited. Howard (1996) notes that even though real options are an integral part of many investment projects, their value has frequently been overlooked when modeling the decision process, and that decisions trees are a natural way to model project flexibility. The relationship between option pricing and decision analysis has been studied by Smith and Nau (1995), who show that options pricing and decision analysis methods give the same results when applied correctly, and propose a method for valuing projects by distinguishing between market risks, which can be hedged by trading securities, and private uncertainties which are project specific risks uncorrelated with the market. Smith and McCardle (1999) illustrate how both option pricing and decision analysis methods can be integrated in the context of a real oil and gas project.

The approach we propose differs from the Smith and Nau approach in the following way. As noted above, their approach relies on distinguishing between market risks and project specific risks. In the context of oil and gas exploration projects, this distinction is often a very natural one, since oil and gas prices are market risks, while the project specific risks may be the probability of a dry hole, or the probability distribution regarding the volume of reserves. In problem contexts such as these, the Smith and Nau approach has a natural appeal. However, there are projects in other industries where the distinction between market risks and project specific risks is either not so clear, or not a meaningful concept. Copeland and Antikorov have suggested an approach to valuing options for these projects, and our methodology provides a practical computational solution for this approach based on the use of binomial decision trees.

The remainder of the paper is organized as follows: Section 2 reviews the traditional approaches to project valuation. Section 3 introduces a decision tree approach to real options modeling based on Copeland and Antikarov's assumptions. In Section 4 we apply the model to solve a sample problem and in Section 5 we conclude with a summary and discussion of further research issues regarding model formulation and solution procedures.

2. Valuation

2.1 Discounted Cash Flow Method (DCF)

With the DCF method the value of a project is determined by discounting the future expected cash flows at a discount rate that takes into account the risk of the project. In complete markets, this discount rate can be inferred by observing the market price of a portfolio of securities that replicate these expected cash flows in all the states of nature and in all future periods. In incomplete markets, there will always be a tracking error due to the difference between the cash flows of the replicating portfolio and those of the project, except in some special cases such as natural resources projects where project cash flows can be perfectly replicated by a portfolio of futures contracts of the commodity and an investment in risk free assets.

As a practical matter, most investment projects are valued using a DCF approach based on the weighted average cost of capital for the firm, or WACC. The determination of the WACC involves the use of the capital asset pricing model (CAPM) to estimate the rate of return required by equity investors from market information regarding stock prices, and this firm-specific information is typically applied to individual investment projects. While the WACC may be an appropriate discount rate for projects that generally mimic the risks associated with the firm as a whole, it may not be appropriate for unusual or innovative investment projects. The practitioner must then use judgment when choosing an appropriate discount rate for the project.

The main criticism of DCF is the implicit assumption that once the firm commits to a project, the project's outcome will be unaffected by future decisions of the firm, thereby ignoring any managerial flexibility the project may have. This managerial flexibility has value, and represents the real options associated with the project.

2.2 Real Options Valuation

Management flexibility is the ability to affect the uncertain future cash flows of a project in a way that enhances its expected returns or reduces its expected losses. Typical project flexibilities include the option to expand operations in response to positive market conditions or to abandon a project that is performing poorly. Management may also have the option to defer investment for a period of time, to temporarily suspend operations, to switch inputs or outputs, to reduce the scale or to resume operations after a temporary shutdown. All of these opportunities represent options on real assets that allow management to enhance the value of the project; thus, they are called real options. The value of these options cannot be determined by the traditional DCF method, but only through option pricing or decision analysis methods.

Option pricing methods were first developed to value financial options. Several pioneering works made the transition from the concepts developed by Black and Scholes (1973) and Merton (1973) for the valuation of financial options to the valuation of options on real assets. Tourinho (1979) used the concept of an option to evaluate a non renewable natural resources reserve under price uncertainty; Brenann and Schwartz (1985) analyzed the optimal operational policy of a copper mine; McDonald and Siegel (1986) determined the optimal timing for investing in a project with irreversible investments with uncertain cost and benefits represented by a continuous time stochastic processes. Dixit and Pindyck (1994) and Trigeorgis (1995) were among the first authors to synthesize several of these ideas.

Traditional option pricing methods require that markets be complete, i.e., that there is a marketed security or a portfolio of securities whose payoffs replicate the payoffs of the project in all states and periods. This is the underlying assumption of much of the work done in the field of continuous time real option valuation (Trigeorgis (1995), Brennan and Schwartz (1985), MacDonald and Siegel (1986)) and allows the determination of the correct discount rate for the project. Although this may be a reasonable assumption for options on financial assets, for most real asset projects no such replicating portfolio of securities exists and markets are said to be incomplete. For this case Dixit and Pindyck (1994) propose the use of dynamic programming using a subjectively defined discount rate, but the result does not provide a market value for the project and its options.

Copeland and Antikarov suggest an alternative discrete time method based on the assumption that the present value of the project without options is the best unbiased estimator of the market value of the project (the Marketed Asset Disclaimer, or MAD assumption). With this assumption, the project itself becomes the underlying asset of the replicating portfolio, thus making the markets complete for the project options. As a result, these options can now be valued with traditional option pricing methods. Another assumption they make is that the variations in the value of the project follow a random walk. While these assumptions are also subject to a number of caveats, we will adopt this point of view for the purpose of this discussion.

2.3 Decision Tree Analysis (DTA) and Risk Neutral Probabilities

Some of the limitations of the DCF method can be overcome with the use of decision tree analysis (DTA). With DTA, managerial flexibility is modeled in discrete time by means of future decision instances that allow the manager to maximize the value of the project conditioned on the information available at that point in time, after several uncertainties may have been resolved.

A naïve approach to valuing projects with real options would be simply to include decision nodes corresponding to project options into a decision tree model of the project uncertainties, and to solve the problem using the same risk-adjusted discount rate appropriate for the project without options. Unfortunately, this naïve approach is incorrect because the optimization that occurs at the decision nodes changes the expected future cash flows, and thus, the risk characteristics of the project. As a consequence, the standard deviation of the project cash flows with flexibility is not the same as that of the project without flexibility, and the risk-adjusted discount rate initially determined for the project without options will not be the same for the project with real options. This fact has caused some authors to wrongly conclude that is inappropriate to use DTA to value real option problems.

However, real option problems can be solved by DTA with the use of risk neutral probabilities. This implies that we can discount the project cash flows at the risk free rate of return and make any necessary adjustments for risk in the probabilities of each state of nature (Smith and Nau, 1995).

An example taken from Copeland and Antikarov illustrates this concept. Suppose there is a two state project with equal chances of cash flows of $170 or $65 one year from now that has a risk-adjusted discount rate of 17.5% and that will cost $115 next year. For obvious reasons, these two states are commonly called the "up state" and the "down state", respectively. The expected present value of the project is [0.5($170) + 0.5($65)] / 1.175 = $100 and the net present value is $2.13 as shown in Figure 1.

Suppose now that the decision to commit to the project can be deferred until next year, after the true state of nature is revealed, and that the risk free rate is 8%. The original discount rate of 17.5% cannot be used because the risk of the project has now changed due to the option to defer the investment decision. On the other hand, a set of risk neutral probabilities for the original project (probabilities that would give the same project value as before when discounting the cash flows at the risk free rate of return) can be determined and used to value the project with the deferral option, since the expected cash flows for both problems are the same ($170 and $65).

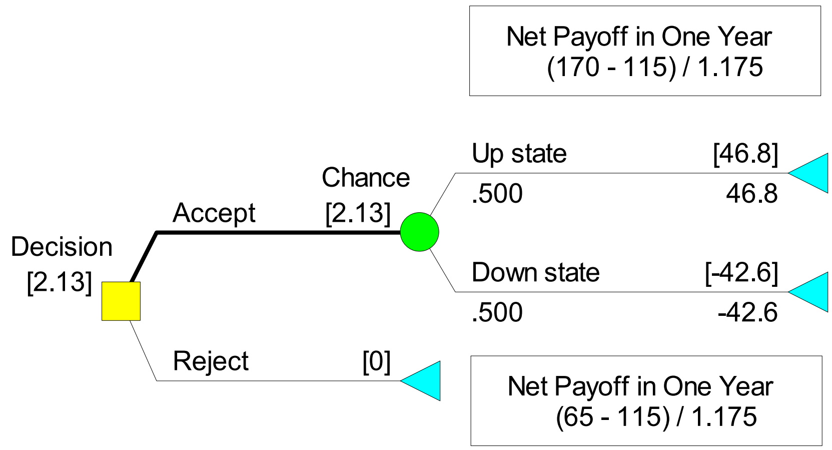

Figure 1– The Project with Objective Probabilities and a Risk-adjusted Discount Rate

While the correct risk-adjusted discount rate of a project with options is difficult to determine due to the effect these options have on the project risk, the risk free rate of return can be readily observed in the market. By switching from objective probabilities to risk neutral probabilities, the project NPV with options can then be estimated even without knowing the correct risk-adjusted discount rate.

In this simple example this can be done by setting the expected present value of the project determined with the objective probabilities and the risk-adjusted discount rate equal to the expected present value of the project with the unknown risk neutral probabilities and the risk free discount rate, and by solving for the risk neutral probability p_r. That is, we would let

$$\$100 = \frac{p_r(\$170) + (1 - p_r)(\$65)}{1 + 0.08}$$

and solve to determine $p_r = 0.41$.

The project with the option to defer has net payoffs of \$170-\$115=\$55 in the up state and zero in the down state as illustrated in Figure 2, as there will be no investment if it is known beforehand that the down state will prevail. The net present value of the project with the option to defer is [0.41(\$55) + 0.59(\$0)] / 1.08 = \$20.86, up from \$2.13. This implies that the value of the option to defer is \$18.73.

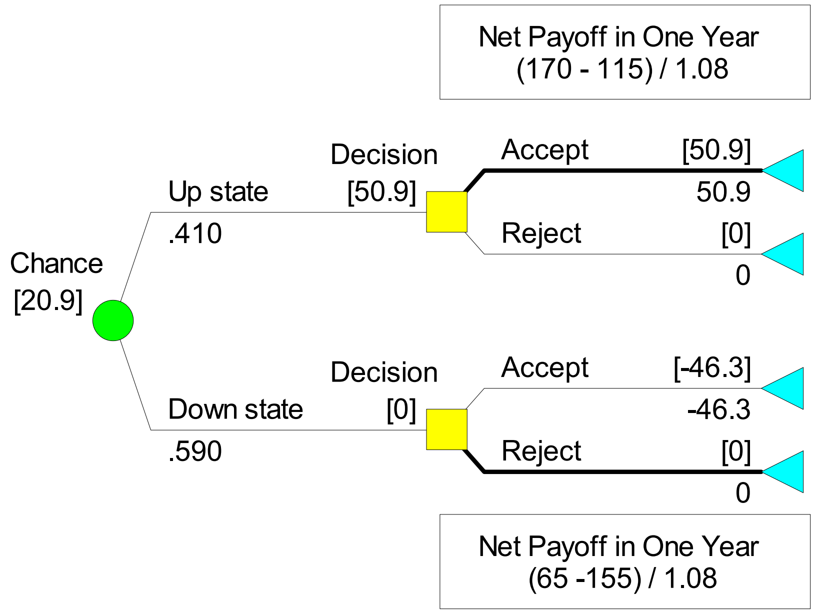

Figure 2 – Project with Risk Neutral Probabilities and a Risk Free Discount Rate

3. The Discrete Time Model for Real Options Analysis

Methods for the evaluation of real options in continuous time have some important practical limitations. Markets are incomplete for the great majority of the projects, and even when the ideal conditions associated with complete markets do occur it may be extremely difficult to determine a market portfolio that has a perfect correlation with the risk of the project. As a result, it may not be practical to find the appropriate discount rate for an individual project.

Suppose we do assume that the WACC is the appropriate discount rate for an individual project without options. However, the existence of managerial flexibility changes the risk of the project since the manager can choose to exercise these options if they increase project value or decrease project losses, so the WACC would not be the appropriate discount rate for the project with options, as illustrated in the previous example.

The model proposed for real options analysis by Copeland and Antikarov (2001) uses two key assumptions to overcome these limitations. The first one is the Marketed

Asset Disclaimer assumption already mentioned in Section 2. The second assumption is that the variations in the project returns follow a random walk.

Let V_i be the value of a project at time period i and V_{i+1}/V_i be its return over the time period between i and i+1. Under the random walk assumption, the logarithm of the return $\ln(V_{i+1}/V_i)$ is normally distributed, and we define v and σ^2 as the mean and variance of this normal distribution. When the time period length tends to zero, this stochastic model can be expressed as an Arithmetic Brownian Motion (ABM) random walk process $d\ln V = vdt + \sigma dz$ where $dz = \varepsilon\sqrt{dt}$ is the standard Wiener process.

The assumption that the distribution of the logarithm of the project returns at any point in time is normal implies that the distribution of the project value at any point in time is lognormal. Accordingly, changes in V_i will be lognormally distributed, and can be modeled as a Geometric Brownian Motion (GBM) stochastic process in the form $dV = \mu Vdt + \sigma Vdz$ where $\mu = v + \frac{1}{2}\sigma^2$. For a discussion of this random walk assumption, see also Hull (1999) and Luenberger (1998).

The importance of this second assumption is the following. A project may involve several uncertainties, which would complicate an effort to model its stochastic process. This assumption allows any number of uncertainties in the model of the project to be combined into one single representative uncertainty, the uncertainty associated with the stochastic process of the project value V, and the parameters of this process can be obtained from a Monte Carlo simulation of the project cash flows. And, as we shall see, a discrete time model using a binary lattice or a binary tree can approximate this continuous time stochastic process. We refer the reader to Copeland and Antikarov for a more thorough discussion of these assumptions.

To illustrate this idea, we assume there is a project that will last m periods, that requires an initial investment I to be implemented, and that generates an expected cash flow C_i, $i = 1,2,...,m$ in each of these periods. For simplicity we assume that the cash flows are paid instantaneously at the end of each time period in a manner analogous to the dividends of a stock. These cash flows represent the dividends distributed by the project where $\delta_i = C_i / V_i$ is the dividend distribution rate and V_i is the pre-dividend value of the project in period i. The project is subject to market uncertainties that will affect its future cash flows, and also has sufficient managerial flexibility to allow an active management to maximize its value during its operational life.

The risk-adjusted discount rate for the project without options is μ. For an internally financed project, this rate may be equal to the firm's WACC, and we assume that the project cash flows will be reinvested in the firm or distributed as dividends after allowing for financing costs. For an externally financed project, where the project is structured as an independent entity with one or more shareholders and non-recourse debt and the cash flows are necessarily distributed as dividends, this rate is equal to the rate of return on equity demanded by the shareholders.

The modeling of the problem will be done in three steps. First the project is analyzed to determine its expected present value at time 0. Next a Monte Carlo simulation is performed with the objective of combining all sources of uncertainty into one single distribution and the stochastic process for the value of the project is defined. The third and last step involves the creation of a binomial tree to model the dynamics of the project value, and of a decision tree with the decision nodes that model the project's real options.

These first two steps are identical to those proposed by Copeland and Antikarov. For the third step we provide an alternative solution methodology based on a binomial tree that offers computational and logical advantages. For completeness, we briefly summarize the first two steps below, and then discuss our proposed modifications of the third step in more detail.

3.1 Modeling with Expected Cash Flows

The present value of the project at time 0, V_0, is determined with the traditional DCF method using a spreadsheet to calculate the expected cash flows $\{C_i,\ i = 1, 2, ..., m\}$ without including the impact of any real options that may exist due to managerial flexibilities associated with the project. These cash flows are then discounted at the risk-adjusted discount rate μ to obtain the present value of the project in each period.

$$V_i = \sum_{t=i}^{m} \frac{C_t}{(1+\mu)^{t-i}} \tag{3.1}$$

The value of the project will decrease in each period due to the payment of dividends, which are assumed to be equal to the cash flows in each period. The dynamics of the evolution of the value of a four period project under conditions of certainty are illustrated in Figure 3.

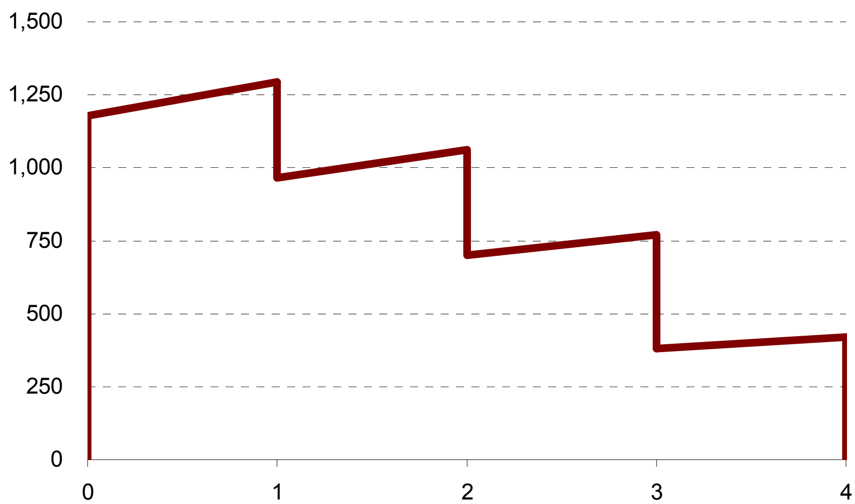

Figure 3 – Project Value Dynamics

3.2 Monte Carlo Simulation

The lognormal distribution of the project's value can be fully defined by the mean and standard deviation of its returns. Note that under the MAD assumption, the present value of the project without options is taken as its market price, as if the project were a traded asset. Assuming that markets are efficient, purchasing the project at this price guarantees a zero NPV and the expected return of the project will be exactly the same as its risk-adjusted discount rate μ. As a result, the mean of the project's returns is exogenously defined. In practice, this risk-adjusted discount rate for the project without options is typically set equal to the firm's WACC, although the analyst may choose a different rate appropriate for each specific project.

The standard deviation, or volatility of the project, can be determined from a Monte Carlo simulation of the ABM process of the returns $d\ln V = vdt + \sigma dz$. The impacts of uncertainties affecting the relevant variables of the project on the returns are determined by simulating each of their stochastic processes, and as a result, the project cash flows become stochastic. Each iteration of the Monte Carlo simulation provides a new set of future cash flows from which a new project value V_1 at the end of the first period is computed using (3.1) with $i = 1$, and a sample of the random variable $\tilde{v}$ can be determined from

$$\tilde{v} = \ln\left(\frac{\tilde{V}_1}{V_0}\right) \tag{3.2}$$

where $E(\tilde{v}) = v$.

A full run of the simulation provides a sample set of the random variable $\tilde{v}$ from which the project volatility is then computed. The volatility σ is defined as the annualized standard deviation of the returns and its computation is straightforward.

3.3 Binomial Lattice

Given the initial project value V_0, the risk-adjusted discount rate μ, and the volatility σ, as previously determined, the value of the project can be modeled in time as a GBM stochastic process by means of a discrete recombinant binomial lattice according to the model of Cox, Ross and Rubinstein (1979) as shown in Figure 4.

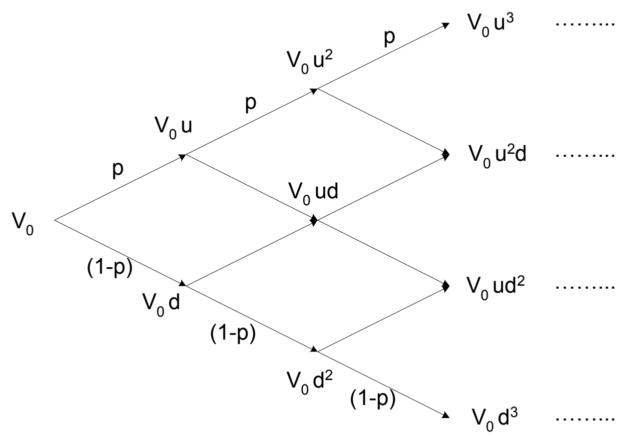

Figure 4– Binomial Lattice

The pre-dividend value of the project in each period and state is given by $V_{i,j} = V_0 u^{i-j} d^j$, where $u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$ are the parameters governing the size of the up and down movements in the lattice. The objective probability of an up movement occurring is $p_\mu = \dfrac{e^{\mu.t} - d}{u - d}$, where *i* = period (*i* = 0, 1, 2, ..., *m*) and *j* = state (*j* = 0, 1, 2, ..., *i*). Note that this objective probability is determined by the value of the risk-adjusted discount rate μ as well as the values of *u* and *d*.

The project pays out dividends in each period in the form of cash flows, and consequently the project value suffers a discontinuity at the time of this distribution. The dividend distribution rate is the fraction of the total project value the cash flows represent in each period. Accordingly, a more accurate representation of the value of the project in time is shown in Figure 5.

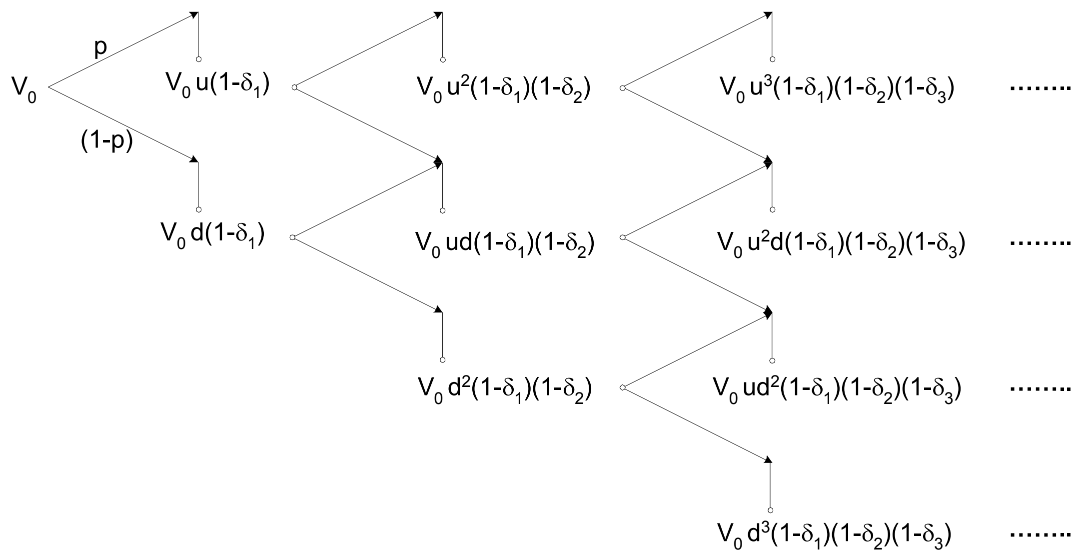

Figure 5– Binomial Lattice with Dividends

The continuous time stochastic process associated with this dividend-paying project is $dV = (\mu - \delta_t)Vdt + \sigma Vdz$, where δ_t is the instantaneous dividend distribution rate at time t. In the discrete time binomial approximation the dividends are explicitly factored into a binomial tree and no further consideration of the dividends is required. To avoid double counting, we will then use the risk-adjusted discount rate μ rather than μ - δ_t as the parameter for the binomial model.

Under uncertainty, the pre-dividend value of the project V_{ij} in period i, state j, is given by the following recursive equation

$$V_{i,j} = V_0\, u^{i-j}\, d^j \prod_{k=1}^{i-1} (1 - \delta_k) \qquad (3.3)$$

The probability $P_{i,j}$ that the value V_{ij} will occur is $P_{i,j} = \binom{i}{j} p^{i-j} (1-p)^j$, where

$\binom{i}{j} = \dfrac{i!}{(i-j)!\,j!}$ is the binomial coefficient.

These relationships provide the results used by Copeland and Antikorov in their solution approach based on the use of a binomial lattice. However, the computational requirements of this approach are cumbersome, due to the requirement of solving for a replicating portfolio at each node in the lattice, and the logic of the binomial lattice when options are included is not transparent.

3.4 Project Decision Tree

In the binomial lattice model, the pre-dividend value of the project in period i and state j is a function of the value V_0 of the project at time zero, of the discount rate μ, of the volatility σ and of the dividend distribution rate δ_i. When the real options of the project are incorporated into the analysis, the binomial lattice (a model of uncertainty) can be transformed into a decision tree (uncertainty plus options).

Modeling the options by determining their impact on the project cash flows is simpler than calculating their impact on the value of the project. An algebraic transformation can be used to value the project as function of a series of artificial cash flows that have the property of guaranteeing that the stochastic process followed by the project value is the same Geometric Brownian Movement determined previously. These cash flows, which we will call pseudo cash flows ($C_{i,j}$), will themselves be a function of the expected cash flows of the project C_i (i = 1, 2, ..., m), of μ and of the parameters u and d of the binomial model. The Marketed Asset Disclaimer assumption assures that markets are complete for the project, and that there exists a unique set of risk neutral probabilities that allow the project to be discounted at the risk free rate of return, as seen in the example in Section 2.3. The solution for the risk neutral probabilities is given by $p_r = \dfrac{e^{rt} - d}{u - d}$, which depends on the risk free discount rate r rather than the risk-adjusted discount rate μ.

The main advantage of this transformation is that it allows the project value function to be expressed in terms of a more basic variable, the project cash flows, providing greater flexibility in the modeling of the real options of the project. We begin by establishing the relation between V_0 and V_1 and the expected cash flow. From equation (3.1),

$$V_i - C_i = (1+\mu)^i \sum_{t=i+1}^{m} \frac{C_t}{(1+\mu)^t} \qquad (3.4)$$

Setting $i = i + 1$ and substituting in (3.1), we obtain

$$V_{i+1} = (1+\mu)\left(V_i - C_i\right) \tag{3.5}$$

There are no cash flows or dividend payments in the initial period ($i = 0$), since the project has not yet been initiated, so $C_0 = 0$. For $i = 0$ we then have

$$V_1 = (1+\mu)V_0 \tag{3.6}$$

The dividend distribution rate is assumed to be constant across states in each period but variable in time, so the cash flows in each period are a fixed proportion of the value of the project in that period and state, as expressed in

$$\delta_i = \frac{C_i}{V_i} = \frac{C_{i,j}}{V_{i,j}} \quad \forall j \tag{3.7}$$

Using (3.3) and (3.7), and calculating $C_{i+1,\,j}$ as function of the previous cash flow $C_{i,j}$, we arrive at

$$C_{i+1,j} = \frac{\delta_{i+1}(1-\delta_i)}{\delta_i}\, u \cdot C_{i,j} \tag{3.8}$$

By analogy we have

$$C_{i+1,j+1} = \frac{\delta_{i+1}(1-\delta_i)}{\delta_i}\, d \cdot C_{i,j} \tag{3.9}$$

and

$$C_{i,j} = \delta_i\, V_0\, u^{i-j} d^{\,j} \tag{3.10}$$

For further simplification, these formulas can also be expressed as functions of the expected cash flows. Substituting $\delta_i = \dfrac{C_i}{V_i}$ and (3.6) in (3.10) we obtain

$$C_{i,j} = \frac{C_i}{V_i} \frac{V_1}{(1+\mu)} u^{i-j} d^j$$

And for $i = 1$,

$$C_{1,j} = \frac{C_1}{1+\mu} \; u^{1-j} d^j \qquad (3.11)$$

Making a similar substitution in (3.9) and (3.10), and using (3.11) we have

$$\begin{cases} C_{i+1,j} = \dfrac{C_{i+1}}{C_i(1+\mu)} \cdot C_{i,j} \; u \\ C_{i+1,j+1} = \dfrac{C_{i+1}}{C_i(1+\mu)} \cdot C_{i,j} \; d \qquad i = 2,3,...,m \qquad j = 0,1,2,...i \end{cases} \qquad (3.12)$$

.

The expression (3.11) provides the pseudo cash flows in the first period of the project. Using this and (3.12), we can obtain the values of the pseudo cash flows in the subsequent periods and states as a function of the pseudo cash flows immediately before, the discount rate μ and the parameters u and d. In other words, (3.12) provides the branch values in each chance node of the decision tree. Since we are using risk neutral probabilities, these cash flows are discounted at the risk free rate to arrive at the present value of the project at time $t = 0$.

Both the binomial lattice and the binomial tree representations of the stochastic process associated with the project values or the pseudo cash flows can be created using the risk-adjusted discount rate μ and the corresponding probability $p_\mu = \frac{e^{\mu.t} - d}{u - d}$ of an up state, or with the risk free interest rate r and the corresponding risk neutral probability $p_r = \frac{e^{rt} - d}{u - d}$. Both approaches will provide the same results for a project with no options, but only the latter can be used for real option valuation.

The use of these pseudo cash flows, rather than the estimates of the project values, allows the easy use of decision trees rather than binomial lattices to evaluate project options. As a result, the evaluation of real options can be carried out conveniently using "off-the-shelf" decision tree software, and allows options to be included in the models using decision nodes that are a natural part of this problem representation. This advantage can best be illustrated with a simple example.

4. Example

We will illustrate this approach to the evaluation of real options with a simple four-period project using commercially available decision analysis software, DPL™. Due to limitations in the software, the decision tree representation is essentially a binary tree augmented by decision nodes, and it is not recombining like a binary lattice. This results in a large tree due to the unnecessary duplication of nodes, but provides a visual interface and a convenient and flexible modeling tool.

The spreadsheet for the example project is shown in Table 1. The risk-adjusted discount rate is assumed to be 10%, the value of the WACC, and the risk free rate is 5%. The first step is simply the computation of the expected value of the future cash flows and the present value of the project at time zero, as illustrated in Table 1.

		0	**1**	**2**	**3**	**4**
Revenue			1000	1080	1166	1260
Variable Cost			(400)	(432)	(467)	(504)
Fixed Cost			(240)	(240)	(240)	(240)
Depreciation			(300)	(300)	(300)	(300)
EBIT			60	108	160	216
Tax	50%		(30)	(54)	(80)	(108)
Depreciation			300	300	300	300
Investment		(1,200)				
Cash Flow		(1,200)	330	354	380	408

PV_0 =	**1,157**	WACC =	10%
Invest =	(1,200)		
NPV =	(43)		

Table 1 – Project Spreadsheet

The present value of $1,157 of the project without options is assumed to be the best estimate of its market value. Since the required investment is $1,200, the project has a negative NPV, which indicates that it should not be implemented.

In this example we assume a single source of uncertainty, the future value of the revenue stream, although other sources of uncertainty could be easily incorporated into the model. Suppose the future project revenues R follow a GBM stochastic process with a mean α_R = 7.70% (which is equivalent to a discrete annual growth of 8.0%) and volatility σ_R = 30%. Next we perform a Monte Carlo simulation on the project cash flows where the future revenues are modeled with these parameters. After a number of iterations we compute the standard deviation of $\tilde{v} = \ln\left(\tilde{V}_1 / V_0\right)$ to obtain an estimate of the project volatility σ = 24.35%. Finally, we assume that the project rate of return is normally distributed, so the project value will have a lognormal distribution at any point in time that may be approximated by a binomial lattice or the corresponding binomial tree.

Next we compute the values of u, d, and the risk neutral probability p_r, according to the formulas defined previously. The pseudo cash flows of the project are computed using formulas (3.12) and (3.13), and the value of the project is determined applying the usual procedures of dynamic programming implemented in a binomial tree, and discounting the expected cash flows at the risk free rate of return. Risk neutral probabilities are used to arrive at the project expected value. The present value obtained with this model is the same as the one calculated with the spreadsheet, as illustrated in Figure 6.

For example, using (3.11), the pseudo cash flow in the upper branch of period 1 is $C_{1,0} = \dfrac{330}{1+0.10} 1.276 = \$382.7$. Discounting this value at the risk free rate r = 5% for one period yields \$364.5, as can be seen in Figure 6. $C_{2,0}$ is computed from (3.12) as $C_{2,0} = \dfrac{354}{330(1+0.10)} (382.7)(1.276) = \$476.1$, which discounted at the risk free rate for two periods yields \$431.9. All other pseudo cash flows can be computed in a similar way.

Note that the values for σ, μ, r and the project expected cash flows C_i can be entered as parameters in a decision tree model, and all the necessary formulae can be incorporated into the tree structure. In effect, tree building can be greatly simplified by developing a standard template for a binary tree for any given number of time periods.

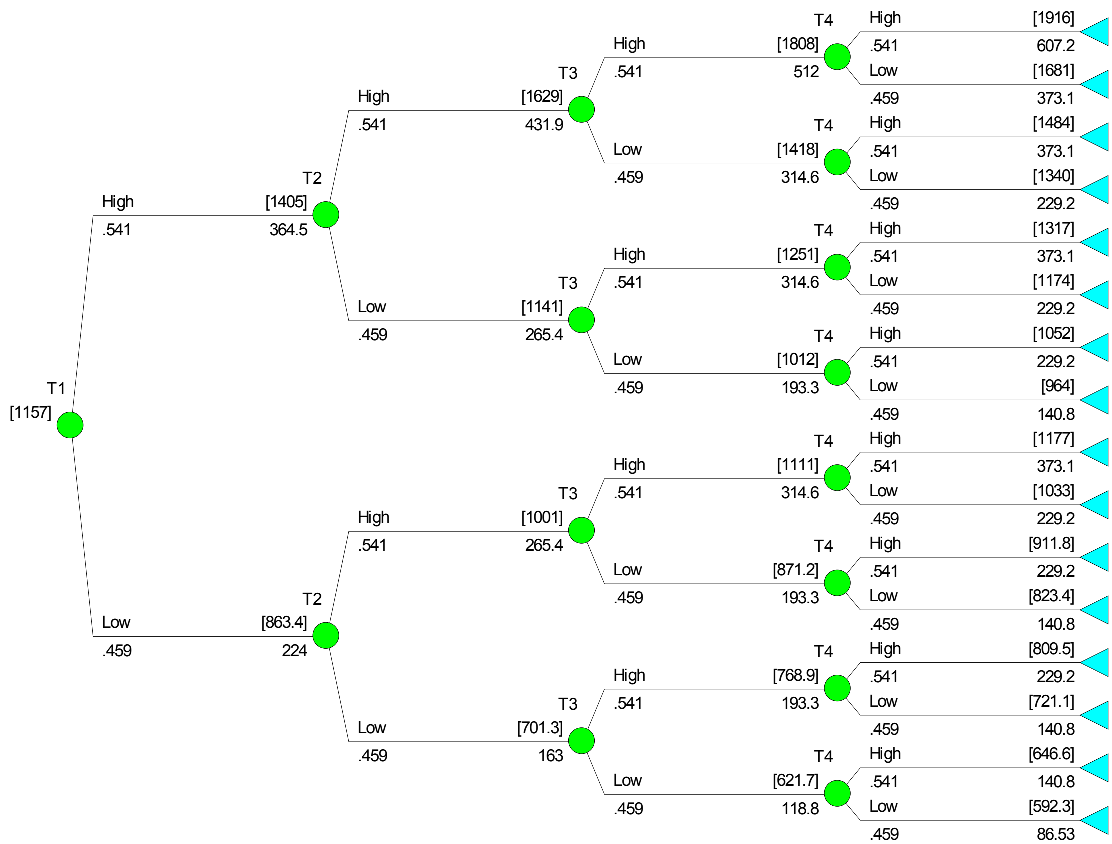

Figure 6 – Project Decision Tree

This binomial tree can now be used to evaluate real options. Suppose the project can be abandoned in the third year of its life for a terminal value of $350. Given the binary tree representation, this option can be evaluated by simply inserting a decision node in time period 3 that models the managerial flexibility that exists in the third year of the project. A new present value for the project is then computed using the same risk neutral probabilities, as illustrated in Figure 7. In some of the states the abandon option will be exercised, and the value of the project with this real option is increased to $1,215.

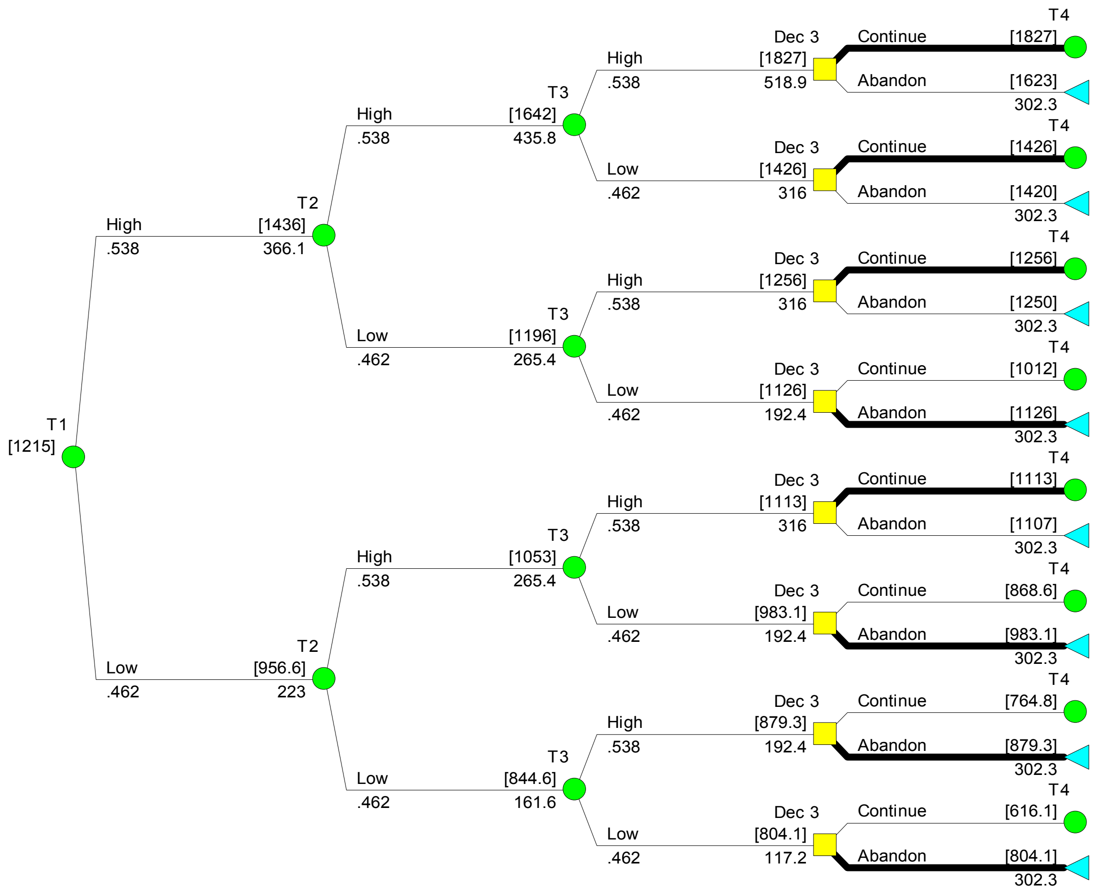

Figure 7 – Decision Tree with Option to Abandon

Once the project's stochastic parameters are determined and the decision tree is structured, additional options can be added with ease. For example, suppose that the option to abandon can also be exercised in year 2, and that there exists an option to expand the project by 30% also in year 2 at a cost of $100. The decision tree model is shown in Figure 8. The project value increases to $1,280, and the expansion option will be exercised in all states of year 2, except one, while the abandon option will continue to

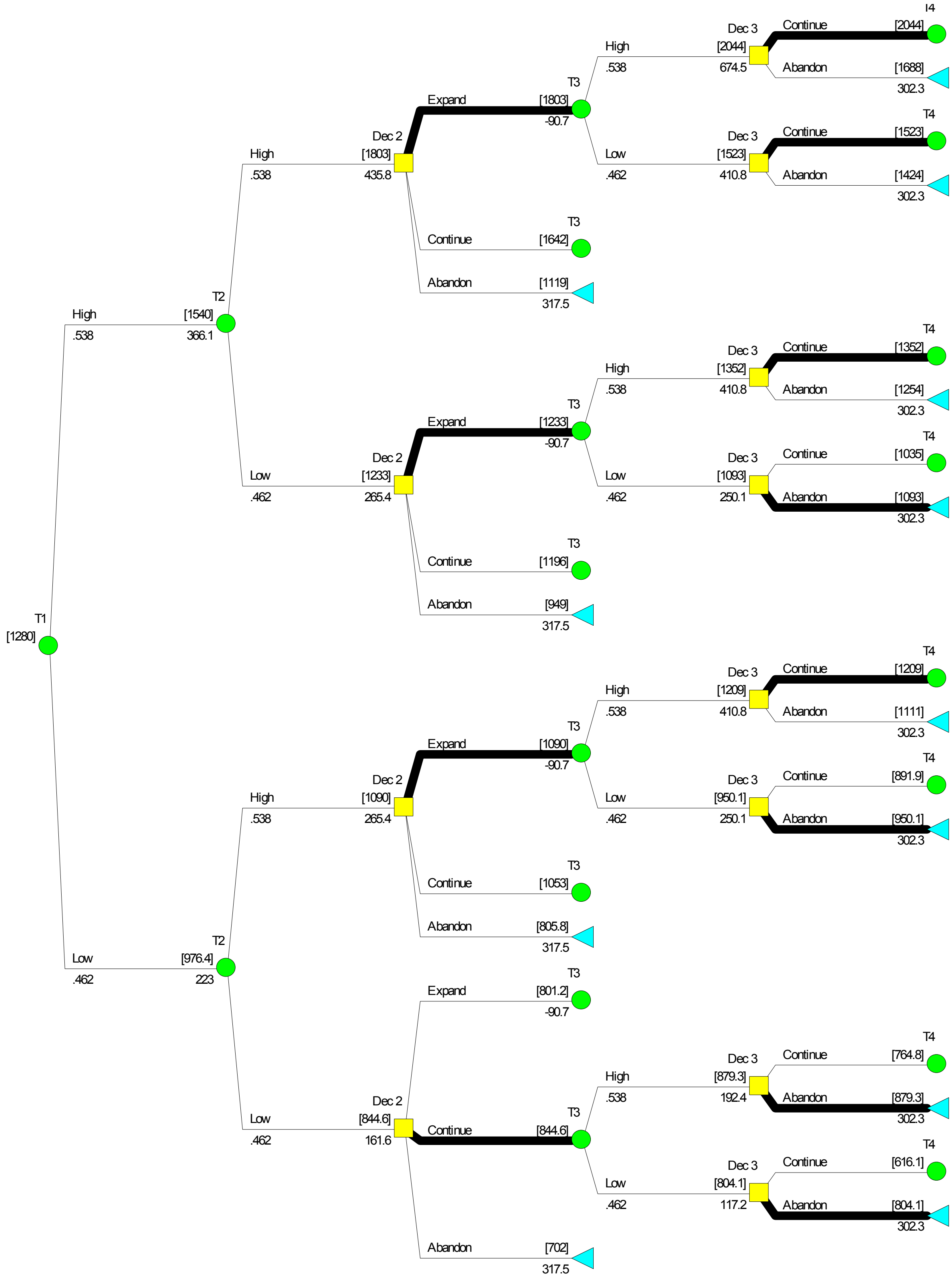

Figure 8 – Decision Tree with Option to Expand and Abandon

be exercised only in year 3, as can be seen by the lines in bold. Additional options and time periods can be added in a straightforward manner.

Even for a simple model such as this one, the decision tree becomes large very quickly. In most practical problems the complexity of the decision tree will be such that full visualization will be impossible. However, even large problems with literally millions of endpoints for the tree can be solved using this approach. Brandao (2002) provides an example of the application of this methodology to the evaluation of options associated with a highway project in Brazil that includes 20 time periods and several different options, resulting in a decision tree with 2×10^9 endpoints that is solved within practical computational times.

5. Conclusions and Recommendations

The method proposed represents a simple and straightforward way of implementing real option valuation techniques using off the shelf decision analysis software. The solution is implemented with decision tree tools that many practitioners currently use. Additional computational efficiencies can be obtained by using specially coded algorithms, although at the cost of having to forgo the simple user interface that decision tree programs such as DPL ™ offer, and the advantage of visual modeling and a logical representation.

Suggested extensions include the implementation of recombining lattice capability in current decision tree generating software to cut down on processing time. While a n period recombining binary lattice has a total of $n(n+1)/2$ nodes, a similar binary tree has $2^{n+1}-1$ nodes, which becomes a significant difference for large values of n. On the other hand, the extension of this model to projects with non-constant volatility (heteroscedasticity) can be easily implemented, whereas the effect of changes in volatility cannot be modeled with a recombining lattice.

Perhaps the primary caveat regarding this methodology for the evaluation of projects with real options relates to the assumptions underlying the Copeland and Antikarov approach itself, since the use of decision trees is simply a computational enhancement of their concepts. The use of the Market Asset Disclaimer as the basis for creating a complete market for an asset that is not traded may lead to significant errors, since the valuation is based on assumptions regarding the project value that cannot be tested in the market place. For example, the appropriate choice of the project discount rate for the project without options is left to the discretion of the analyst, and the use of WACC may not be appropriate for all projects. Therefore, it is important to realize that this thorny issue is not resolved by this methodology.

This approach is also based on the valuation of the project without options, which may not be a meaningful concept in the context of some projects, such as those in the pharmaceutical industry, where there are natural options associated with the development of new drugs. It is simply not clear how one would value a project related to the development of a new product in this industry without explicitly recognizing these options. To the extent that such hypothetical projects without options are not representative of typical projects in the industry, then the WACC may not be an appropriate risk-adjusted discount rate for them. In such circumstances, there may be no useful guidelines for choosing the risk-adjusted discount rate for the project without options.

Also, the notion that the project returns will vary according to a random walk is a very strong assumption. In some cases this may be considered a reasonable assumption, but other investment projects may include "lumpy" or discrete events that make this assumption untenable, and at best, it may be considered only an approximation. Therefore, we would suggest that this approach should be considered for the valuation of projects with real options only after a careful consideration of these assumptions in the context of specific applications, as it may not be applicable to all situations.

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